Lesson 5-8 Rational Zeros Theorem

Rational Zeros Theorem

If $P(x)$ is a polynomial function with integral coefficients, then every rational zero of $P(x) = 0$ is of the form $\frac{p}{q}$ where $p$ is a factor of the constant term and $q$ is a factor of the leading coefficient.
Example!

List all possible rational zeros

a) \( f(x) = 3x^4 - x^3 + 4 \)

\[ \frac{P}{Q} \]

\( P: \) factors constant factors of 4
\( Q: \) factors leading coefficient factors of 3

\( \pm 1, \pm 2, \pm 4, \pm 1, \pm 3 \)

\( \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3} \)

b) \( f(x) = x^4 + 7x^3 - 15 \)

\( P: \) factors of 15
\( Q: \) factors of 1

\( \pm 1, \pm 3, \pm 5, \pm 15, \pm 1 \)

\( \pm 1, \pm 3, \pm 5, \pm 15 \)
Example 2
The volume of a rectangular solid is 1120 cubic feet. The width is 2 feet less than the height and the length is 4 feet more than the height. Find the dimensions of the solid.

\[ h \]

\[ W = h - 2 \]

\[ L = h + 4 \]

\[ h(h^2 - 2h + 4h - 8) = 1120 \]

\[ h^3 + 2h^2 - 8h = 1120 \]

\[ h^3 + 2h^2 - 8h - 1120 = 0 \]

\[ p: \text{factors of 1120} \]

\[ q: \text{factors of 1} \rightarrow \pm 1 \]

\[ \pm 1, \pm 2, \pm 4, \pm 5, \pm 7, \pm 8, \pm 10, \pm 14, \pm 16, \pm 20 \]

\[ \pm 1120, \pm 560, \pm 280, \pm 224, \pm 160, \pm 140, \pm 112, \pm 80, \pm 70, \pm 56 \]

\[ \pm 28, \pm 40, \pm 32, \pm 35 \]

Put equation in calculator

\[
\begin{array}{c}
10 \\
10 \\
8 \\
14
\end{array}
\]
Example 3
Find all the zeros of \( f(x) = x^4 + x^3 - 19x^2 + 11x + 30 \)

\[ \begin{array}{c}
P: \text{factors of 30} \\
\pm 1, \pm 2, \pm 3, \pm 5, \pm 6 \\
\pm 10, \pm 15, \pm 30 \\
\text{Put in calculator} \\
X = 2
\end{array} \]

Synthetic div
\[
\begin{array}{c|cccc}
2 & 1 & 1 & -19 & 11 & 30 \\
& & 2 & 6 & -26 & -30 \\
\hline
& 1 & 3 & -13 & -15 & 0
\end{array}
\]

\[ x^3 + 3x^2 - 13x - 15 \]

\[ \text{Put in calculator} \]
\[ X = -1 \]

\[
\begin{array}{c|cccc}
-1 & 1 & 3 & -13 & -15 \\
& & -1 & -2 & 15 \\
\hline
& 1 & 2 & -15 & 0
\end{array}
\]

\[ x^2 + 2x - 15 \]

factors -15 | add 2
\[ -3 \ 5 \]

\[ (x-3)(x+5) = 0 \]

\[ X = 2 \]
\[ X = -1 \]

\[ X = 3 \]
\[ X = -5 \]
Page 370

10-16 even
20-26 even
30-36 even